Introduction to Probability Theory
Problem set #3

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January 5, 2011

Abstract

Solutions to the problem set number three of the subject Statistics, Monte Carlo Methods and Data Processing - Master in Astrophysics, Particle Physics and Cosmology (Universitat de Barcelona).

1 Length determination

For a set of ten measurements with a resolution of 1 mm it is possible to define several different estimators such as adding up all the measurements and divide it by 10 (average value) $\widehat{\mu} = \frac{1}{10}(x_1 + ... + x_{10})$ with variance $V(\widehat{\mu}) = \frac{\sigma^2}{n} = \frac{1^2}{10} = 0.1$

On the other hand, a unique measurement with a resolution of 0.2 mm ($\widehat{\mu} = measurement$) will present a variance of $V(\widehat{\mu}) = \frac{\sigma^2}{n} = \frac{0.2^2}{1} = 0.04$. Therefore, it is preferable to use this unique measurement.

2 Maximum likelihood estimator

Sample data: $\{x_i\} = 1...N$ drawn according to a probability distribution density $f_\theta(x) = \frac{1}{\theta}e^{-x/\theta}$ where x is positive real.

Through the maximum likelihood it is possible to deduce:

$$\ln(L) = -\sum_i \frac{x_i}{\theta} - \ln(\theta) \Rightarrow \frac{d\ln(L)}{d\theta} = \sum_i \left(\frac{x_i}{\theta} - \frac{1}{\theta}\right) \tag{1}$$

Obtaining the estimator:

$$\widehat{\theta} = \frac{1}{n}\sum_i x_i \tag{2}$$

If we consider the probability $Proba[X \leq r]$ where $r > 0$ and X is distributed according to f_θ:

$$f'_\theta(x) = \frac{1}{\theta} e^{-x/\theta} \left(1 - e^{-r/\theta}\right) \tag{3}$$

Then the maximum likelihood estimator will be:

$$\widehat{\theta} = \frac{1}{n} \sum_i x_i + \frac{1}{n} \sum_i \frac{r \cdot e^{-r/\widehat{\theta}}}{1 - e^{-r/\widehat{\theta}}} \tag{4}$$

References

[1] Iblisdir, S. (2010). *An Introduction to Probability Theory.* Universitat de Barcelona.